

Problem :

Standing on the sidewalk, you observe a car parked far down the road. The driver turns on the headlights, and you measure the speed of that emanating light to be, "C", that is $C = 3E8$ m/s.

Now that same car is approaching you at (hypothetically) the speed of light. The driver turns on the headlights... and once again you measure the speed of the light emanating from the headlights.

Do you determine that this second lightspeed measurement gives $2C$?

Solution:

In our everyday existence, the motions of objects are governed by Galilean transformations. At high speeds (compared to the speed of light) however, the motion of objects is governed by the Lorentz transformations.

At slow speeds the Lorentz transformations degenerate to become the Galilean transformations to within a very good approximation. However one cannot obtain the Lorentz transformations from the Galilean transformations when considering high speed objects.

If we applied the Galilean transformation to our current problem, here's what we'd have...

$$V_o = V_C + V_L$$

Where, $V_o =$ The speed of light as determined by you, in the problem,

$V_L =$ The speed of the oncoming car,

$V_C =$ The speed of light as determined by the car's driver.

In our problem, the Galilean transformation gives,

$$V_o = C + C ; \quad \text{or,} \quad V_o = 2C .$$

Experimental evidence however, supports the concept that the Galilean transformations are only a good approximation to the more valid, Lorentz transformations. The Lorentz transformation for speeds is given by,

$$V_o = \frac{V_C + V_L}{1 + \frac{V_C \cdot V_L}{C^2}}$$

In this last formula, convince yourself that if V_L is tiny, compared to the speed of light, then the rightmost term in the denominator is effectively, zero. In this case the Lorentz formula looks just like the Galilean formula.

Apply the Lorentz formula to the problem at hand. The speed you determine for the light emanating from the moving car is,

$$V_o = \frac{C + C}{1 + \frac{C \cdot C}{C^2}} \rightarrow V_o = \frac{2C}{1+1} \rightarrow V_o = C$$

The car moving at speed, C, will produce light which you also determine is moving at speed, C.

Don't get the wrong impression, though. Mother Nature did not get up one fine morning, and ponder whether she'll go with Galilean transformations or Lorentz transformations.

She just made a characteristic of spacetime, that the speed of light is the same for all inertial observers (i.e., those moving at constant speeds relative to one another).