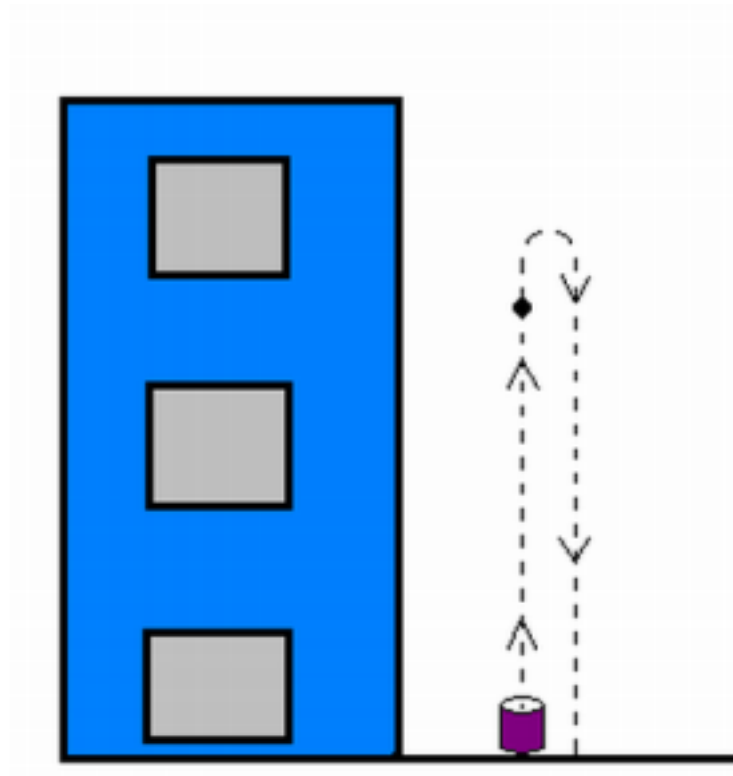


Problem :

**A ball is shot vertically upward from the ground. Its initial speed is 128 ft/sec.**

**a) How many seconds later, will it be at a height of 32 feet?**

**b) What is the maximum height reached by the ball?**



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Solution :

Solve this general equation :  $S = S_0 + V_0T + \frac{1}{2}gT^2$

Where,  $S$  = height of ball at time,  $T$  .

$S_0$  = initial height of the ball

$V_0$  = initial speed of the ball

$g$  = acceleration due to gravity,  $32ft/sec^2$

Taking the upward direction as (+) and downward as (-) then the Initial speed and the acceleration due to gravity are in opposite

directions. Therefore the equation must be modified to reflect this...

$$S = S_0 + V_0T + \frac{1}{2}gT^2 \quad \text{becomes,}$$

$$S = S_0 + V_0T - \frac{1}{2}gT^2$$

$$32 = 0 + 128 \bullet T - \frac{1}{2}(32) \bullet T^2 \quad \text{and this simplifies to,}$$

$$16 \bullet T^2 - 128 \bullet T + 32 = 0$$

$$T^2 - 8 \bullet T + 2 = 0, \quad \text{solve the quadratic equation,}$$

$$T = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(2)}}{2(1)} \quad \rightarrow \quad T = 0.25 \text{ sec.}, \text{ and } T = 7.75$$

The upward moving ball reaches 32 feet elevation in about **0.25 sec.** The quadratic equation gives us two answers. What does the other answer (i.e., 7.75 sec) mean?

At the maximum height, the ball's instantaneous speed is zero, because it then must start downward. That is,

$$\frac{dS}{dT} = 0 \quad \rightarrow \quad \frac{dS}{dT} = 0 + V_0 - gT \quad (\text{taking the 1}^{\text{st}} \text{ derivative})$$

This latter equation tells us the time (T) when the ball reaches maximum height. Substitute this time into the very first equation To get the maximum travel height...

$$\frac{dS}{dT} = 0 + V_0 - gT$$

$$0 = 0 + 128 - 32 \bullet T_{\text{max}} \quad \rightarrow \quad T_{\text{max}} = 4 \text{ sec. to reach maximum.}$$

Now use,

$$S = S_0 + V_0T - \frac{1}{2}gT^2$$

$$S_{\max} = 0 + 128(4) - 16(4^2) \quad \rightarrow \quad S_{\max} = 256 \text{ feet.}$$

The ball takes 4 sec. to reach the pinnacle of travel, and reaches a maximum height of **256** feet.

**NOTE :**

How does one obtain the starting equation,  $S = S_0 + V_0T + \frac{1}{2}gT^2$  ?

It all begins by recognizing the fact that the force ( $F_g$ ) on the ball depends on its mass, and on the earth's effective gravitational acceleration:

$F_g = mg$  but since  $F_g = ma$ , where ( $a$ ) = acceleration, we have...

$ma = mg$  or,  $a = g$ . (The assumption that inertial mass and gravitational mass are one in the same, is called the "Equivalence Principle"... and that's why we can cancel the masses in that equation)

But for our problem we choose a sign convention where up is (+) and where down is (-). Therefore,  $a = -g$ .

Since the acceleration is the *first* derivative of the speed (or, the *second* derivative of the distance) we have,

$$a = -g \quad \rightarrow \quad \frac{dV}{dT} = -g \quad \rightarrow \quad dV = -g \bullet dT \quad , \quad \text{integrate...}$$

$$\int_{V_0}^V dV = -g \int_0^T dT \quad \rightarrow \quad V - V_0 = -g(T - 0) \quad \rightarrow \quad V = V_0 - gT$$

Now use the definition of speed... namely,  $V = \frac{dS}{dT}$  ,

$$V = V_0 - gT$$

$$\frac{dS}{dT} = V_0 - gT \quad \rightarrow \quad dS = (V_0 - gT)dT \quad , \quad \text{integrate...}$$

$$\int_{S_0}^S dS = V_0 \int_0^T dT - g \int_0^T T dT \quad S - S_0 = V_0 T - g \cdot \frac{T^2}{2} \quad , \quad \text{or,}$$

$$S = S_0 + V_0 T - \frac{1}{2} g T^2$$