

Problem :

Assume that all income to the federal treasury comes from taxation, and assume the validity of the following statement:

**If we decrease the excessive tax rates, then federal revenue increases.**

Which of the following statements is **necessarily** also valid?

(1):

If we do not decrease the excessive tax rates, then federal revenue does not increase.

(2):

If federal revenue increases, then we decreased the excessive tax rates.

(3):

If federal revenue does not increase, then we did not decrease the excessive tax rates.

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Solution :

Statement (1) is the *inverse* of the original statement.

Statement (2) is the *converse* of the original statement.

Statement (3) is the *contrapositive* of the original statement.

The original (given) statement is called a conditional statement: ( IF 'A' THEN 'B' )

***Given the validity of the original conditional statement, the contrapositive statement must have the same validity... therefore statement (3) above, is also necessarily valid.***

You can verify this conclusion using a truth table:

Let "p" represent "we decrease the excessive tax rate"

Let "q" represent "federal revenue increases"

Let " ~p " represent the negation, "we do **not** decrease the excessive tax rate"

Let " ~q " represent the negation, "federal revenue does **not** increase"

Our original statement is the conditional statement... If p then q.

This conditional is abbreviated,  $p \rightarrow q$  .

It is important to remember that the only way a conditional statement can be false, is if (p) is true while (q) is false...  $\{T \rightarrow F\}$  ; otherwise, the conditional statement is has a 'true' value.

p	q	$\sim p$	$\sim q$	$p \rightarrow q$ (original)	$\sim p \rightarrow \sim q$ (inverse)	$q \rightarrow p$ (converse)	$\sim q \rightarrow \sim p$ (contrapos.)
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

From this table, see that the **original** statement and the **contrapositive** statement have the **same** truth values, while the **inverse** and **converse** statements do not.