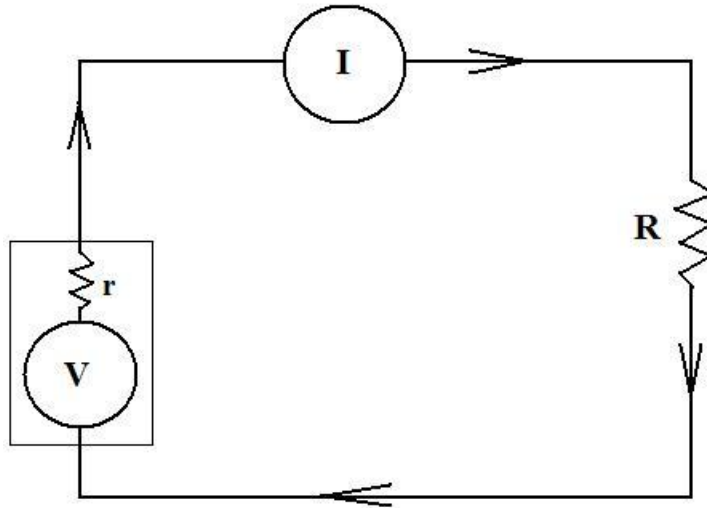


Problem :

A simple DC circuit has a load consisting of a light bulb, having an adjustable resistance (R), and a power supply having a constant voltage (V) and constant internal resistance (r).



(1) :  
Show that the maximum power output from the bulb occurs when ( R = r ).

(2) :  
Show that for this *maximum power* at (R), the efficiency of this circuit is *only 50 %* .

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Solution :

Write the voltage drops around the circuit;

$$V = (I \cdot r) + (I \cdot R) \quad \rightarrow \quad V = I(r + R) \quad \rightarrow \quad I = \frac{V}{(r + R)}$$

The power at R is given by;

$P = I^2 R$  ; Substitute the current term above. You do the work, and get...

$$P = V^2 \cdot \frac{R}{(r + R)^2}$$

For maximum power, set  $\frac{dP}{dR} = 0$  . How do we know we'll get a maximum?

$$\frac{dP}{dR} = V^2 \cdot \frac{(r+R)^2(1) - R(2)(r+R)(1)}{(r+R)^4} \rightarrow \frac{dP}{dR} = V^2 \left[ \frac{r^2 - R^2}{(r+R)^4} \right]$$

For  $\frac{dP}{dR} = 0$  , it is required that  $r^2 = R^2 \rightarrow r = R$  .

So, maximum wattage at the bulb resistance (R) occurs when the bulb resistance is the same as the internal resistance of the battery.

The efficiency is given by,

$$\text{Efficiency} = \frac{I^2 R}{I^2 r + I^2 R} \rightarrow \text{Efficiency} = \frac{R}{r + R}$$

Substitute the earlier result for maximum power at (R)...  $r = R$  .

$$\text{Efficiency} = \frac{R}{R + R} \rightarrow \text{Efficiency} = \frac{R}{2R}$$

$$\text{Efficiency} = \frac{1}{2} \rightarrow 50\% , \text{ when operating at maximum power at (R).}$$

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