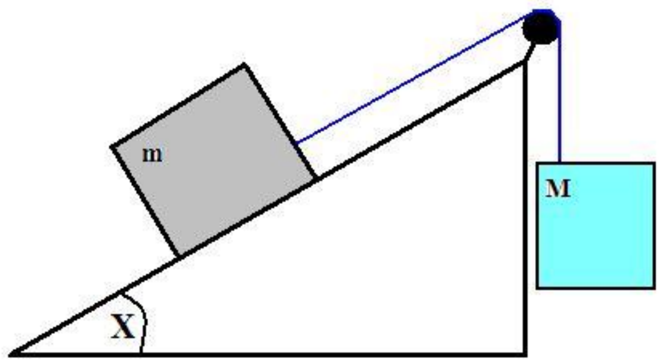
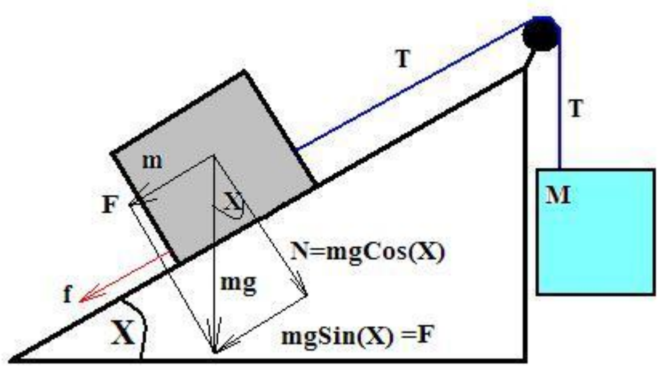
Problem :

For given masses (m), (M), angle (X), and coefficient of kinetic friction (μ) on the surface of the incline, find a general expression for the acceleration of the two blocks in this diagram :



Solution :

Here are the forces on the blocks…



This diagram is drawn with the assumption that the block (M) moves downward and the block (m) slides up the incline. If this is what happens, then the acceleration (a) will be positive. If we compute a negative acceleration, that means the block (m) slides down the incline, and the block (M) moves upward.

(f) = force due to friction; $f = \mu N \quad \rightarrow \quad f = \mu mgCos(X)$

(F)= component of weight (mg) directed down the incline; $F = mgSin(X)$

(T)= tension (force) in the cord connecting (m) and (M).

Based on the assumption built into the diagram, the acceleration of (m) can be described in terms of the forces on (m):

$$ma = T - F - f \;; \quad \rightarrow \quad ma = T - mgSin(X) - \mu mgCos(X) \qquad \textbf{(1)}$$

The acceleration of (M) is the same as for (m) since the cord is not stretching. The acceleration Of (M) is described from…

$$Ma = Mg - T \qquad \textbf{(2)}$$

Add equations **(1)** and **(2)** to eliminate the tension (T). Simplify the resulting equation…

$$(m + M)a = Mg - mgSin(X) - \mu mgCos(X)$$

On the right side, factor out the (g). Divide both sides by (m), and solve for acceleration (a)…

$$a = g \bullet \left[\frac{\left(\frac{M}{m}\right) - Sin(X) - \mu Cos(X)}{1 + \left(\frac{M}{m}\right)} \right]$$

Case #1: let $M = m$ and $X = 45^{\circ}$ and $\mu = 0.57735$ and $g = 9.8\, m/\sec^2$
Compute the magnitude and direction of the acceleration (a) of the blocks.

You should get… $a = -0.57\, m/\sec^2$; the minus sign indicates (m) slides *down* the incline

and (M) moves upward. With (a) known, you can get the tension in the cord by substituting numbers into either of equations **(1)** or **(2)**.

Case #2 : Repeat Case #1, except set the angle of inclination to $X = 30^{\circ}$

You should get… $a \approx 0.0$; Provide an interpretation of this result.