Problem:

45 ml of 0.25 M Acetic Acid is titrated with 55 ml of 0.15 M Sodium Hydroxide. What is the pH of the resulting solution?

Solution :

Recognize that the reaction is a neutralization, involving a weak acid and a strong base.

H(Ac) + NaOH → Na(Ac) + H(OH)

Notice that the acetate radical $(C_2H_3O_2)^-$ is abbreviated by (Ac).

When the reaction is completed, let's see if *both* reactants get used up…

#moles of H(Ac) reactant = $(0.045L)\bullet(0.25moles/L)\rightarrow 0.01125moles$

#moles of Na(OH) reactant = $(0.055L)\bullet(0.15moles/L)\rightarrow 0.00825moles$

Since there is a 1:1 relationship between the reactants in the above reaction, then the titrant, Na(OH), will be used up first, and there will be an excess of H(Ac) in the beaker, when the reaction reaches the end.

Excess H(Ac) remaining = 0.01125 - 0.00825 → 0.0030 moles of H(Ac).

Also, since the Na(OH) is limiting, we will have formed 0.00825 moles of water, in addition to the 45 ml of aqueous acid solution and 55 ml of aqueous base solution.

The extra volume of water (assuming room temperature) is…

Extra ml water = $(0.00825moles)\bullet\left(18\frac{gm}{mole}\right)\bullet\left(\frac{1}{1}\frac{ml}{gm}\right)\rightarrow 0.15ml$

In the above equation, the middle term is the molecular weight of water, and the last term is the inverse of the density of water- at room temperature. Pay attention to the cancellation of the units, to see that the result is a volume of water.

The final volume of liquid in the beaker is… 45 + 55 +0.15 = 100.15 ml of H(OH).

The concentration of the excess H(Ac) in the beaker, is…

$$[HAc] = \frac{0.0030}{0.10015}\frac{moles}{L} \rightarrow 0.0300\frac{moles}{L}$$

But also in the beaker, is the fully ionized salt, Na(Ac)…

$$Na(Ac) \leftrightarrow Na^{+} + (Ac)^{-}$$

Since in the original neutralization (see above), the Na(Ac) is also in a 1:1 proportion with the reaction-limiting Na(OH), then we will have formed 0.00825 moles of Na(Ac). And since this salt is fully ionized in aqueous solution, we will have the following final concentration of acetate ions from the Na(Ac)…

	$Na(Ac)$	$\leftrightarrow$	Na^{+}	+	$(Ac)^{-}$
#moles :	.00825		.00825		.00825

Concentration of $(Ac)^{-}$ ions from the Na(Ac) = $\frac{0.00825}{0.10015}\frac{moles}{L} \rightarrow 0.0824\frac{moles}{L}$

Since the excess H(Ac) is a weak acid, it will not fully ionize in the water of the beaker…

	$H(Ac)$	$\leftrightarrow$	H^{+}	+	$(Ac)^{-}$
$\frac{\#\,moles}{L}$:	.0300 – X		X		X + .0824

In the above assessment, we account for the formation of an undetermined amount of hydrogen ions dissociated from the weak acid. The same concentration (X) of acetate ions will be formed by the dissociation of the weak acid, but, we also have to account for all the acetate ions in the beaker, due to the full dissociation of Na(Ac) into ions (that's the 0.0824, above).

The ionization constant for acetic acid is given by… $K_a = 1.8 \times 10^{-5}$

$$K_a = \frac{[H^{+}] \bullet [(Ac)^{-}]}{[H(Ac)]}$$

$$1.8 \times 10^{-5} = \frac{[X] \bullet [X + .0824]}{[.0300 - X]}$$

Let's assume that X << .030… that will avoid us working a laborious and unnecessary quadratic equation solution. We'll check our assumption when we find (X)…

$$1.8\times10^{-5}=\frac{[X]\bullet[.0824]}{[.0300]}\quad;\quad\text{solve this for (X)… show that ;}$$

$$[X]=6.553\times10^{-6}\,\frac{moles}{L}\quad;\quad\text{but from above,}\quad[X]=[H^{+}].$$

Compare this result for (X) to the assumption (X<<0.03)… the assumption is a valid one.

Now that we have the hydrogen ion concentration, we can find the pH of the resultant solution in the beaker…

$$pH=-Log[H^{+}]$$

$$pH=-Log[6.553\times10^{-6}]$$

$$pH=-\{Log(6.553)+Log(10^{-6})\}$$

$$pH=-\{Log(6.553)+(-6)\bullet Log(10)\}$$

$$pH=6-0.816$$

$$pH=5.2$$

The solution is on the acid side of pH = 7. Play with the numbers a little… did we, for example, really need to account here, for the water generated in the neutralization reaction?